VI. *On Captain* Parry's *and Lieutenant* Foster's *experiments on the velocity of sound. By Dr.* Gerard Moll, *Professor of Natural Philosophy in the University of Utrecht. Communicated by Captain* Henry Kater, *V.P.R.S.*

Read January 17, 1828.

During Captain Parry's winter residence at Port Bowen, in 1824—1825, experiments were instituted on the velocity of sound. As, probably, investigations of that sort will not frequently be made at such low temperatures, it appears not uninteresting to compare the results obtained, with the theoretical formula of the late lamented Laplace, and also with other experiments made under different circumstances. With this view I caused my assistant M. Simons to make the calculations of which I am about to render an account.

Captain Parry's and Lieut. Foster's experiments were made at Port Bowen, in 73° 13′ 39″ N. and 88° 54′ 55″ W. of Greenwich. The distance of the brass six-pounder from the station of the observers was trigonometrically determined by Captain Parry to be 12892,96 English feet, and by Lieut. Foster 12892,82 feet; the mean being 12892,89 feet. Time was measured by pocket chronometers held close to the observers' ears. The direction of the gun was S. 71° 48′ E. The table of the experiments is repeated, to avoid the necessity of reference.

Date.	Barom.	Therm.	Wind.		Weather.	No. of guns fired.	Interval in Seconds between the Flash and Report.			Rate of travelling per 1″ in feet.
			Direction.	Force.			Parry.	Foster.	Mean.	
1824.	Engl. inch.	Fahr.					″	″	″	
Novemb. 24	29,841	− 7°	E.S.E.	light	overcast	5	12,3525	12,430	12,3912	1040,49
December 8	29,561	− 9	N.N.E.	squally	very clear	6	12,331	12,5266	12,4288	1037,34
1825.										
January 10	30,268	−37	E.S.E.	light	clear	4	12,5889	12,4700	12,5290	1029,01
February 7	29,647	−24,5	N.E.	light	very clear	6	12,639	12,6167	12,6278	1020,99
17	29,598	−18		calm	overcast	6	12,372	12,440	12,406	1039,25
21	29,735	−37,5		calm	overcast	6	12,8167	12,7067	12,7617	1010,28
March 2	30,398	−38,5	easterly	light	overcast	6	12,640	12,780	12,710	1014,39
22	30,258	−21,5	westerly	light	clear, and fine	6	12,400	12,7167	12,5583	1026,64
June 3	30,118	+33,5	easterly	light	very clear	6	11,7333	11,744	11,7387	1098,32
4	30,102	+35	S.E.	strong	clear	6	11,5889	11,4733	11,5311	1118,10

As it is suggested by the observers themselves, that the experiment of the 4th of June was influenced by a strong wind, blowing in the direction of the base, I shall not take the result of that day into the account. From the mean of all the other observations, we have the velocity of sound in one second equal to 1035,19 English feet, at a barometric pressure of 29,936 English inches, the temperature being $-17°,72$ FAHR.

LAPLACE's theoretical formula, for the computation of the velocity of sound, is

$$V = \sqrt{\frac{gp}{D}} \times \sqrt{\frac{c'}{c}} \; *$$

V being that velocity.
p the barometric pressure.
D the density of the air.
$\frac{c'}{c}$ the ratio between the specific heat of air, at a constant pressure and at a constant volume.

Taking the French metre equal to 39,38255 English inches †, as results from English and French comparisons, we have 1035,19 feet = 315,4597826 metres; 29,936 inches = 0,76013 metres; $-17°,72$ FAHR. $= -27°,1$ Centig.

According to BIOT's and ARAGO's experiments, and taking the dilatation of mercury by heat as determined by DULONG and PETIT, we have the weight of a cube centimetre of mercury at 0° Centig. 13,596152 grammes. The same philosophers found the weight of a cube centimetre of dry atmospheric air, under a pressure of 760 millimetres, and at a temperature of 0° Centig., at Paris, in latitude 48° 50′ 14″, equal to 0,001299541 grammes. To compute from these data the weight of a cube centimetre of atmospheric air at Port Bowen at the temperature of 0° Centig., and the pressure of 760 millimetres, we must multiply the number 0,001299541 by the ratio of the intensity of gravity at Paris and at Port Bowen.

Let the intensity of gravity at Port Bowen be g, the latitude $l = 73° \; 13' \; 39''$;

* LAPLACE, Annales de Chimie et de Physique T. III. p. 238. Poisson sur la vitesse du son, Annales de Chimie et de Physique Mai 1823, p. 5.

† I found afterwards that it had been better to have adopted Captain KATER's comparison of the metre with English inches, as given in the Phil. Trans. for 1818, p. 103.

at Paris g', the latitude $l = 48° \ 50' \ 14''$; and in latitude 45° (g); g' being equal to 9,8088 metres: We have then

$$g = (g)\ (1-0{,}002837 \cos 2\ l)\ ;\ \text{and}\ g' = (g)\ (1-0{,}002837 \cos 2\ l'):$$

$$\frac{g}{g'} = \frac{1-0{,}002837 \cos 2\,l}{1-0{,}002837 \cos 2\,l'} = \frac{1{,}002364503}{1{,}000378864};$$

$$\text{and } g = g' \left\{ \frac{1{,}002364503}{1{,}000378864} \right\} = 9{,}8088 \left\{ \frac{1{,}002364503}{1{,}000378864} \right\}.$$

Thus the weight of a cube centimetre of dry air, at a pressure of 760 millimetres, and at the temperature 0° Centig., is at Port Bowen

$$\frac{0^{\text{v}}{,}001299541 \times 1{,}002364503}{1{,}000378864} = 0{,}0013021206 \text{ grammes};$$

and the density of the air at Port Bowen, under a pressure of 760 millimetres, and in temperature 0°,0, is

$$\frac{0{,}0013021206}{13{,}596152} = \frac{1}{10441{,}545}$$

Consequently at an atmospheric pressure p, and temperature t, the density of air, at Port Bowen, is

$$D = \frac{p}{10441{,}545 \times 0{,}760\,(1+0{,}00375\,t)};\ \text{and as } t = -27°{,}62,$$

$$D = \frac{p}{10441{,}545 \times 0{,}760\,(1-0{,}00375 \times 27{,}62)}.$$

According to the experiments of Messrs. GAY-LUSSAC and WELTER, the ratio between the specific heat of air, under a constant pressure, and that under a constant volume, $\frac{c'}{c} = 1{,}3748$.

Substituting the above values in the formula

$$V = \sqrt{\frac{gp}{D}} \times \sqrt{\frac{c'}{c}} \quad \text{we have}$$

$$V = \sqrt{9{,}82827 \times 10441{,}55 \times 0{,}760 \times 0{,}896425} \times \sqrt{1{,}3748}$$

$$= 310{,}0305696 \text{ metr.} = 1017{,}72 \text{ English feet.}$$

The general result of Captain PARRY's and Lieut. FOSTER's experiments gives for the velocity of sound . . . . 315,426 metr. = 1035,19 feet.
The theoretical calculation gives . . . . 310,031 metr. = 1017,72 feet.

Difference between calculation and experiment . . . . 5,395 metr. = 17,47 feet.

Now, if we take the mean of the experiments made at Port Bowen the 17th and 21st February 1825, when the weather was calm, we have the velocity by experiment, 1024,765 feet = 312,249446 metr., at a barometric pressure of 0,75328 metr., and a temperature of $-27°,75$ FAHR., or $-33°,2$ Centig.

Thus we have $D = \frac{p}{0,760 \times 0,8755 \times 10441,55}$; which being substituted in the foregoing expression, we have the velocity of sound,

$$V = \sqrt{9,82827 \times 10441,55 \times 0,8755 \times 0,76} \times \sqrt{1,3748} = 306,39072 \text{ metr.}$$

The velocity by experiment . . . . = 312,249446 metr.

Difference between computation and experiment, on the 17th and 21st February 1825 . . . = 5,858726 metr.

Again, comparing the experiments of the 22nd March and 3d June with theory, we may hope to have some compensation for the effect of wind. On the 22nd March the wind was westerly, and on the 3d June easterly. Both experiments were made at a temperature comparatively high. The velocity of sound was, by the mean of the experiments of these two days, 1062,48 English feet, or 323,741284 metr. The barometric pressure was 30,188 inches, or 0,76653 metr. The temperature was $+6°$ FAHR., or $-14°,4$ Centig.

Calculating with these data, we have the velocity of sound, on the 22nd March and 3d June, by theory . . . . 318,488009 metr.
By the experiments . . . . 323,741284 metr.

Difference by the experiments of 3d June and 22nd March . . . . 5,253275 metr.

Difference by the experiments of 17th and 22nd February . . . . 5,858726 metr.

Difference by all the experiments but the last . . . 5,395 metr. or 17,47 feet.

In the experiments which I made with Dr. VAN BEEK, and which are recorded in the Phil. Trans. for 1824, we had the following differences between calculation and experiment.

By the experiments of 27th June, 1823, difference 4,92 metr. = 16,147 feet.
By the experiments of 28th June, 1823, difference 4,24 metr. = 13,916 feet.

The final inference to be drawn appears, that in those high latitudes, the uncertainty of the data on which the calculations are founded, is somewhat greater than at higher temperatures. LAPLACE himself says of these differences, " qu'elles paraissent être dans les limites des petites erreurs dont cette expérience, et les élémens de calcul, dont j'ai fait usage, sont encore susceptibles."

Perhaps it might occur to some, that Captain PARRY and Lieut. FOSTER ought to have observed the hygrometer; but I think the objection unfounded. I believe it may be shown, that even supposing the air saturated with aqueous vapour, it could, at those low temperatures, have no influence on the velocity of sound.

Let the tension of aqueous vapour in the atmosphere be T. According to M. GAY-LUSSAC's experiments, the density of aqueous vapour is $\frac{10}{16}$ of the density of dry air. Thus we have

$$D = \frac{p - \frac{3}{8}\,T}{10441{,}55 \times 0{,}76\,(1 + 0{,}00375\,t)}$$

which being substituted in the theoretical formula, it becomes

$$V' = \sqrt{\frac{g \,.\, p \,.\, 10441{,}55 \times 0{,}76\,(1 + 0{,}00375\,t)}{p - \frac{3}{8}\,T}} \times \sqrt{\frac{c'}{c}}$$

We calculated Captain PARRY's and Lieut. FOSTER's experiments by the formula

$$V = \sqrt{10441{,}55 \,.\, g \,.\, 0{,}76\,(1 + 0{,}00375\,t)} \times \sqrt{\frac{c'}{c}}$$

thus we have

$$V' = V \,.\, \sqrt{\frac{p}{p - \frac{3}{8}\,T}}.$$

Now supposing the degrees of the centesimal thermometer under 100° or the boiling point to be N, and T_N the tension of aqueous vapour at that temperature, in a space saturated with moisture; we have, by a formula deduced by M. Biot from M. Dalton's experiments*

$$\text{Log. } T_N = \bar{1}.8819493 - 0{,}01537271116\, N - 0{,}0000673241\, N^2 + 0{,}00000003377\, N^3$$

Captain Parry's experiments were made at the temperatures of −27°, −33°, and −14° Centig. Thus we must successively take N equal to 127, 133, and 114; and calculating on those several suppositions, we have the tension of aqueous vapour, at

−27° C. .... 0,000816121 = T
−33° C. .... 0,000542991 = T′
−14° C. .... 0,002010982 = T″

The general result of all Captain Parry's experiments, at a temperature of −27° Centig. and barometric pressure of 0,76013 metr., was V=310,0305696 metres.

we have $T = 0{,}00081621$; $\frac{3}{8}\,T = 0{,}000306045375$; $p - \frac{3}{8}\,T = 0{,}759823954625$ metr.

and thus $V' = 310{,}0304696 \text{ metr.} \times \sqrt{\frac{0{,}76013}{0{,}759824}} = 310{,}0929624$ metr.

Thus, even supposing that the air in Captain Parry's and Lieut. Foster's experiments was as moist as possible, the difference in the velocity of sound at such low temperatures could be only 0,0623921 metr., or about 2 inches $\frac{4}{10}$.

Calculating on the same supposition, the experiments of the 17th and 21st February, and of the 22nd March and the 3d of June 1825; and supposing the air as moist as can be: we have, for the 17th and 21st of February 1825,

$$V' = 306{,}43213 \text{ metr.}$$

and thus the velocity altered by 0,04141 metr. or about $1\frac{7}{10}$ inch in 1″.

For the experiments of the 22nd March and the 3d June 1825, the difference is somewhat greater. For we have then

$$V' = 318{,}64481 \text{ metr.}$$

and the velocity is altered by 0,156801 metr. or $6\frac{2}{10}$ inches.

* Biot, Traité de Physique, T. I. p. 277.

At all events, the effect produced by moisture, under the circumstances in which the experiments at Port Bowen were made, is so trifling, that it may safely be neglected altogether.

I shall now proceed to compare the experiments of the northern navigators with those of Dr. VAN BEEK and myself, and shall reduce for that purpose the Port Bowen experiments to what they would have been at the temperature of 0° C. or 32° F.

Taking V″ as the velocity of sound at 0° C., and D the density of air at that temperature, we have

$$V'' = \sqrt{\frac{g \cdot p}{D}} \times \sqrt{\frac{c'}{c}}$$

Let V be the velocity at a temperature t, we have again

$$V = \sqrt{\frac{g\,p\,(1+0{,}00375\,t)}{D}} \times \sqrt{\frac{c'}{c}}$$

wherefore

$$V'' = \frac{V}{\sqrt{1+0{,}00375\,t}}$$

The mean of Captain PARRY's and Lieut. FOSTER's experiments, excepting those of the 4th of June 1825, give

V = 315,42597826 metr.; the temperature $t = -27°{,}62$ Centig.

Whence V″	= 333,15 metr.
In the same manner, calculating V″ for the experiments of the 17th and 21st of February 1825, we have	333,71
And for those of the 22nd of March and 3d of June . . .	332,85
Dr. VAN BEEK's and my experiments give	332,05
Messrs. STAMPFER and VON MYRBACH in 1822 in Germany .	333,25
Messrs. ARAGO, MATHIEU and BIOT, in France	331,05
Mr. BENZENBERG in Germany	333,70
Messrs. EPINOZA and BAUZA in Chili	356,14
Dr. OLINTHUS GREGORY in England	335,14
The French Academicians in 1738	332,93

Thus the differences between Captain PARRY's experiments and ours, when both are reduced to the same temperature of 0° is $1^{m}\cdot 1$, $1^{m}\cdot 7$, and $0^{m}\cdot 8$. These differences will be deemed very small, if we consider, that between our own experiments the difference was 0,66 metr., and that between those of the members of the French Board of Longitude, there were still more considerable differences.

These results on the whole appear very satisfactory; and the near agreement of experiments, made under circumstances so widely different, must lead us to suspect that whatever difference still remains between the results of computation and observation, must be ascribed, in a great measure, to some imperfection of the theoretical formula, and not to any fault or neglect of the observers.

Another conclusion may be fairly drawn from the coincidence of the results obtained by Captain PARRY and his friend, with those of many other observers. I mean the great accuracy with which Captain PARRY's proceedings were conducted. Our own experiments were made under every favourable circumstance; in the middle of summer, in a place where nothing was wanted which could give ease and comfort to the observers. Captain PARRY and Lieut. FOSTER operated in a dreary climate, partly in a polar winter, far from every thing which could make their task easy, and above all at a temperature of which it is frightful to think. Besides this, Captain PARRY had many other important matters at the time committed to his care. Our observations, and those of many other observers, were a party of pleasure; theirs, a painful drudgery. And still with every thing thus in our favour, provided with all the means which we could think of, we could do no better than he did in the most inhospitable climate of the globe. It appears that Captain PARRY's experiments are as accurate as those of any other observer. This equality I should consider as a proof of superiority, considering the peculiar circumstances under which the experiments at Port Bowen were made.